of gyration of our noncircular section, suggests that these corrections would also be negligible for data with much narrower tolerances than we are able to specify. Our data cannot directly support this conclusion for it is possible that our large tolerances result in part from these effects.

Within the context of the above considerations, we believe our experiment to be a reasonable compromise as regards both the use of the plane elastic wave formalism in extended media for our finite sized specimens, and the use of predominantly energy-refracting modes in determining the elastic constants. Judging from the topological fitting procedure presented, we estimate that the values given are accurate to within about 5%.

APPENDIX I

In this section, we outline the general procedure used to calculate the energy flow components and present the expressions obtained for the 45° $(l,m,n::0,1/\sqrt{2},1/\sqrt{2})$ and 135° $(l,m,n::0,-1/\sqrt{2},1/\sqrt{2})$ propagation directions.

The ith Cartesian component of energy flow, P_i, is given by Love[19] as the negative of the scalar product of the component of the stress tensor on the surface normal to the ith direction, $\mathbf{T}_i$, with the particle displacement velocity $\dot{\mathbf{u}}$:

$$P_i=-\mathbf{T}_i\cdot\dot{\mathbf{u}}. \tag{A1}$$

The displacement

$$\mathbf{u}=p\mathbf{A}\exp(j(\omega t-\mathbf{K}\cdot\mathbf{r})) \tag{A2}$$

has components u_i where i runs from 1 to 3 corresponding to the x, y, z or x_1, x_2, x_3 directions. $\mathbf{A}$, $\mathbf{K}$, and $\mathbf{r}$ are in this order the particle displacement eigenvector of unit magnitude, the wave propagation vector, and the field point vector, and have components A_i, K_i, x_i. p is the scalar amplitude of the displacement; $\mathbf{T}_i$ has components X_{ij}, $j=1,2,3$. These are related in the usual way to the strains e_{rs} through the stiffness constants by

$$X_{ij}=c_{ijrs}(1+\delta_{rs})e_{rs}/2 \tag{A3}$$

summed for $r,s=1,2,3$; δ_{rs} is the Kronecker delta. In terms of the displacements,

$$e_{rs}=\left(\frac{\partial u_r}{\partial x_s}+\frac{\partial u_s}{\partial x_r}\right)\Big/(1+\delta_{rs}). \tag{A4}$$

For a particular mode g, the components of displacement, written as

$$u_i{}^g=p^gA_i{}^g\exp[j(\omega t-\mathbf{K}^g\cdot\mathbf{r})], \tag{A5}$$

are substituted into (A1) and (A4), and the result of substituting (A4) into (A3) in turn put into (A1). We finally obtain

$$P_i{}^g=\frac{-(p^g\omega)^2}{2v_g}c_{ijrs}A_j{}^gA_r{}^gl_s{}^g, \tag{A6}$$

where $l_s{}^g$, the cosine of the angle between $\mathbf{K}^g$ and the s coordinate axis, is l, m, or n for the gth mode, as $s=1,2$, or 3. This expression is valid for crystals of any symmetry. It differs from Waterman's[18] Eq. (5.1) in that it is written directly in terms of the stiffness constants. (The four-index notation is reduced to the two-index notation in the usual way: $ij\to a$, $rs\to b$; $11\to 1$, $22\to 2$, $33\to 3$, $23=32\to 4$, $13=31\to 5$, $12=21\to 6$.)

Our results for $\mathbf{K}$ with direction cosines $(0,m,n)$ are the following: For $g=10$, $A^{10}=(1,0,0)$ for antimony and bismuth and

$$P_1{}^{10}=0, \tag{A7}$$

$$P_2{}^{10}=\frac{-(p^{10}\omega)^2}{2v_{10}}(mc_{66}+nc_{14}), \tag{A8}$$

$$P_3{}^{10}=\frac{-(p^{10}\omega)^2}{2v_{10}}(mc_{14}+nc_{44}). \tag{A9}$$

For $g=9$ and 11, we have $\mathbf{A}^9=(0,\ 0.7513,\ 0.6599)$, $A^{11}=(0,\ -0.6599,\ 0.7513)$ for antimony and $(0,\ 0.7696,\ 0.6385)$ and $(0,\ -0.6358,\ 0.7696)$ for bismuth; $m=n=1/\sqrt{2}$.

$$P_1{}^g=0, \tag{A10}$$

$$P_2{}^g=\frac{-(p^g\omega)^2}{2v_g}([mc_{11}-nc_{14}]A_2{}^gA_2{}^g+[-m2c_{14}+n\{c_{44}+c_{13}\}]A_2{}^gA_3{}^g+mc_{44}A_3{}^gA_3{}^g), \tag{A11}$$

$$P_3{}^g=\frac{-(p^g\omega)^2}{2v_g}([-mc_{14}+nc_{44}]A_2{}^gA_2{}^g+m[c_{44}+c_{13}]A_2{}^gA_3{}^g+nc_{33}A_3{}^gA_3{}^g). \tag{A12}$$

The appropriate $P_i{}^g$ for propagation in the $(0,-1/\sqrt{2},1/\sqrt{2})$ direction follow from (A7)–(A12) by replacing $\pm m$ with $\mp m$, and the g indices 9, 10, 11 with 12, 13, and 14, respectively. The unit eigenvectors are $\mathbf{A}^{13}=(1,0,0)$, $\mathbf{A}^{12}=(0,\ -0.8625,\ 0.5060)$, $\mathbf{A}^{14}=(0,\ 0.5060,\ 0.8625)$ for antimony and $(1,0,0)$, $(0,\ -0.8421,\ 0.5393)$, $(0,\ 0.5393,\ 0.8421)$ for bismuth.

In the cases discussed, $P_1\equiv 0$, a result to be expected when the excitation does not disturb the mirror symmetry of the medium. The energy-flux deviation angle from the Z or X_3 axis, α, is $\tan^{-1}P_2/P_3$.

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